বিদ্যাসাগর বিশ্ববিদ্যালয়

VIDYASAGAR UNIVERSITY

Question Paper

B.Sc. Honours Examinations 2022

(Under CBCS Pattern)

Semester - IV

Subject : MATHEMATICS

Paper : C 9 - T

Multivariate Calculus

Full Marks : 60

Time : 3 Hours

Candidates are required to give their answers in their own words as far as practicable.
The figures in the margin indicate full marks.

1. Answer any ***five*** questions : 2×5=10

(a) Distinguish between double integral and repeated integral.

(b) Show that $\vec{\nabla}|\vec{r}|^n = n|\vec{r}|^{n-2}\,\vec{r}$, where $\vec{r} = xi + yj + zk$.

(c) Show that $\vec{\nabla}\Phi$ is a vector perpendicular to the surface $\Phi(x, y, z) = c$, where c is a constant.

(d) Write down the formula for the evaluation of length of a curve. Justify it.

(e) Show that $\lim\limits_{(x,y)\to 0} \dfrac{x - y^2}{x^2 + y^2}$ does not exist.

P.T.O.

(f) Find the equation of the tangent plane to the surface $f(x,y) = x^2 + y^2 + \sin xy$ at the point (0, 2, 4).

(g) Find the surface area of a sphere by using surface of revolution.

(h) If $\vec{A}$ and $\vec{B}$ are irrotational, show that $\vec{A} \times \vec{B}$ is irrotational.

2. Answer any ***four*** questions : 5×4=20

(a) State and prove the Schwartz's theorem for the equality of f_{xy} and f_{yx} at some point (*a, b*) of the domain of definition of *f* (*x, y*).

(b) Express $\int_0^{\frac{\pi}{2}} dx \int_0^{\cos x} x^2\, dy$ as a double integral and evaluate it.

(c) Prove $\vec{\nabla} \times \left(\vec{F} \times \vec{G}\right) = \vec{F}\left(\vec{\nabla}.\vec{G}\right) - \vec{F}.\vec{\nabla}\vec{G} + \vec{G}.\vec{\nabla}\vec{F} - \vec{G}\left(\vec{\nabla}.\vec{F}\right)$, where $\vec{F}$ and $\vec{G}$ are differentiable vector function.

(d) Find $\iint_R f(x,y)\, dxdy$, over the region *R* bounded by $x = y^{\frac{1}{3}}$ and $x = \sqrt{y}$ where $f(x,y) = x^4 + y^2$.

(e) What is the maximum directional directional derivative of $g(x,y) = y^2 e^{2x}$ at (2, –1) and in the direction of what unit vector does it occur?

(f) Let *f* and *g* be twice differentiable functions of one variable and let $u(x,t) = f(x+ct) + g(x-ct)$ for a constant *c*. Show that $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$.

3. Answer any ***three*** questions : 10×3=30

(a) (i) Find the minimum value of $x^2 + y^2 + z^2$ subject to the constaint $ax + by + cz = 1\left(a \neq 0, b \neq 0, c \neq 0\right)$.

(ii) Show that $f(x,y,z) = \left(x^2, y^2, z^2\right)^{-\frac{1}{2}}$ is harmonic. 8+2

(b) (i) Let z be a differentiable function of x and y and let $x = r\cos\theta, y = r\sin\theta$,

Prove that $\frac{\partial^2 z}{\partial r^2} + \frac{1}{r}\frac{\partial z}{\partial r} + \frac{1}{r^2}\frac{\partial^2 z}{\partial \theta^2} = \frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2}$. 7

(ii) Prove that $f(x,y) = \begin{cases} \frac{x^3 + y^3}{x - y}, & x \neq y \\ 0, & x = y \end{cases}$ is not continuous at (0, 0). 3

(c) (i) Prove that $\iiint \frac{dxdydz}{x^2 + y^2 + (z-2)^2} = \pi\left(2 - \frac{3}{2}\log 3\right)$, extended over the sphere $x^2 + y^2 + z^2 \leq 1$.

(ii) Using a double integral, prove that the relation $B(m,n) = \frac{\Gamma m \Gamma n}{\Gamma(m+n)}$,

m, n > 0. 5+5

(d) (i) Verify Stoke's theorem for the function $\vec{F} = x^2 i - xyj$ integrated round the square in the plane $z = 0$ and bounded by the lines $x = 0$, $y = 0$, $x = a$, $y = a$.

(ii) Prove that $\iint \left[2a^2 - 2a(x+y) - (x^2 + y^2)\right] dxdy = 8\pi a^4$, the region of integration being the interior of the circle $x^2 + y^2 + 2a(x+y) = 2a^2$. 6+4

(e) (i) Evaluate $\iint_S \overline{A}\,.\,\hat{n}\,ds$; $\overline{A} = 2yi - zj + x^2 k$ over the surface S of the bounded by the parabolic cylinder $y^2 = 8x$, in the first octant bounded by the plane $y = 4$ and $z = 6$. 7

(ii) Find the directional derivative of $f(x,y) = 2x^2 - xy + 5$ at (1, 1) in the direction of unit vector $\left(\frac{3}{5}, -\frac{4}{5}\right)$. 3